Surname	Centre Number	Candidate Number
Other Names		2

GCE A LEVEL

1300U30–1

MATHEMATICS – A2 unit 3
PURE MATHEMATICS B

2 hours 30 minutes

ADDITIONAL MATERIALS
In addition to this examination paper, you will need:
• a Formula Booklet;
• a calculator.

INSTRUCTIONS TO CANDIDATES
Use black ink or black ball-point pen. Do not use gel pen or correction fluid.
You may use a pencil for graphs and diagrams only.
Write your name, centre number and candidate number in the spaces at the top of this page.
Answer **all** questions.
Write your answers in the spaces provided in this booklet. If you run out of space, use the additional page(s) at the back of the booklet, taking care to number the question(s) correctly.

INFORMATION FOR CANDIDATES
The maximum mark for this paper is 120.
The number of marks is given in brackets at the end of each question or part-question.
Sufficient working must be shown to demonstrate the **mathematical** method employed.
Answers without working may not gain full credit. Unless the degree of accuracy is stated in the question, answers should be rounded appropriately.
You are reminded of the necessity for good English and orderly presentation in your answers.

For Examiner's use only		
Question	Maximum Mark	Mark Awarded
1	11	
2	12	
3	6	
4	6	
5	8	
6	8	
7	5	
8	5	
9	4	
10	3	
11	10	
12	7	
13	11	
14	11	
15	7	
16	6	
Total	120	

Reminder: *Sufficient working must be shown to demonstrate the **mathematical** method employed.*

1. (a) Differentiate each of the following functions with respect to x.

(i) $(e^{-x} + x^2)^3$

[2]

(ii) $e^{2x} \sin 3x$

[3]

(iii) $\frac{\cos 2x}{x^2 + 1}$

[3]

(b) A function is defined implicitly by

$$2x - 3x^2y^3 = 5$$

Find $\frac{dy}{dx}$ in terms of x and y.

[3]

2. (a) Find all values of x in the range $0 \leq x \leq 2\pi$ satisfying the equation

$$\cos 2x + \sin x = 0$$

Give your answers in terms of π.

[6]

(b) Find all values of θ in the range $0^o \leq \theta \leq 360^o$ satisfying the equation

$$2 \sec^2 \theta - 5 = \tan \theta$$

[6]

3. Given that θ is small, find an estimate for the solution of the equation

$$\frac{\cos \theta}{1 + \sin 2\theta} = 0.8$$

[6]

4. (a) Using the trapezium rule with 5 ordinates, find an approximate value for

$$\int_0^{\frac{\pi}{3}} \sec^2 x \, dx$$

Show your working and give your answer correct to three decimal places.

[4]

(b) Hence, deduce an approximate value for

$$\int_0^{\frac{\pi}{3}} (\tan^2 x + 2)\, dx$$

[2]

5. A curve is defined by $f(x) = x^3 - 6x^2 + 12x - 5$.

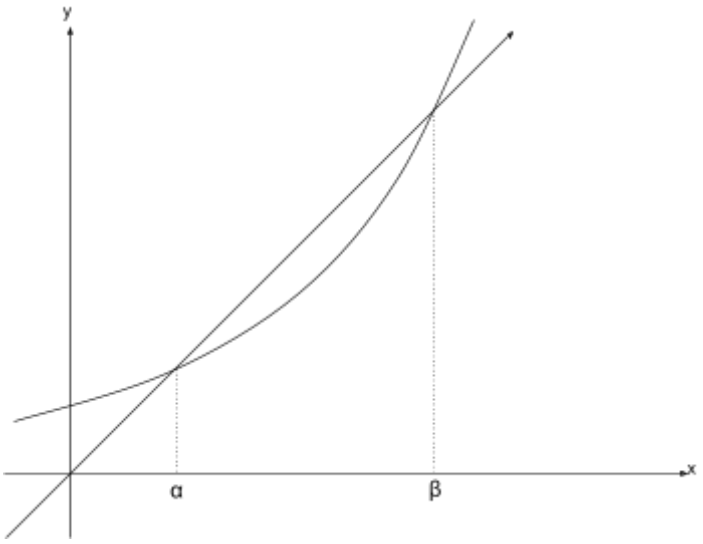
(a) Find the coordinates of the stationary point and determine its nature.

[6]

(b) Determine the range of values of x where the curve is convex.

[2]

6. The diagram below shows a sketch of the curve with equation $y = e^x$ and the straight line with equation $y = 3x$.



The straight line intersects the curve at points where $x = \alpha$ and $x = \beta$. Approximate values for α and β are 0.62 and 1.5 respectively.

(a) The iterative formula $x_{n+1} = \frac{1}{3} e^{x_n}$ is used to find a more accurate approximation for α. Taking $x_0 = 0.62$, use the iterative formula to obtain the values of x_1, x_2 and x_3. Give your answers correct to three decimal places.

[2]

(b) The Newton–Raphson method is used to find a more accurate approximation for β.

(i) Taking $f(x) = e^x - 3x$ and starting with $x_0 = 1.5$, apply the Newton–Raphson method once to obtain a second approximation, x_1. Give your answer correct to three decimal places.

[3]

(ii) Show that x_1 is the value of β correct to three decimal places.

[2]

(iii) Explain why the Newton–Raphson method fails if the starting value in the approximation of β is ln3.

[1]

7. The first and last terms of an arithmetic series are -3 and 61 respectively. The sum of all of the terms is 957. Determine

(a) the number of terms,

[2]

(b) the common difference,

[2]

(c) the middle term.

[1]

8. By algebraic division, or otherwise, simplify the expression

$$\frac{x^4 + 5x^3 - 7x^2 - 29x + 30}{3(x+5)(x+3)(x-2)}$$

[5]

9. Use proof by contradiction to show that there are no integers x and y for which $3x + 15y = 2$ [4]

10. (a) Sketch the graph of $y = \sec x$ for $0 \leq x \leq \alpha$, where $0 < \alpha < \frac{\pi}{2}$ [1]

(b) Explain why $\int_0^{\alpha} \sec x \, dx > \alpha$ [2]

11. Find $\int_1^2 \frac{5x + 2}{(x+1)^2(2x-1)}\,dx$ giving your answer in the form from lna +b, where a, b are constants. [10]

12. Expand $\frac{2-x}{\sqrt[3]{(1-2x)}}$ in ascending powers of x up to and including the term in x^3.

State the range of values of x for which the expansion is valid.

[7]

13. A curve is defined parametrically by

$$x = 2t^3 + 1, \qquad y = 3t^2$$

(a) Show that the gradient of curve at the point with parameter p is 1/p

[4]

(b) Find the equation of the normal to the curve at the point with parameter t, where t = 1.

[4]

(c) Determine the equation of the tangent to the curve at the point with parameter t, where t = 0 [3]

14. (a) Use integration by parts to find the exact value of

$$\int_0^{\frac{\pi}{2}} x^2 \cos x \ dx$$

[6]

$$\int \frac{\ln x}{x\,(1+\ln x)^2}\,dx$$

(b) Use the substitution $u = 1 + \ln x$ to find

[5]

15. A function f is defined by $f(x) = \frac{9}{(x-3)^2}$, where $x > 3$

(a) Determine the range of f

[2]

(b) Find an expression for $f^{-1}(x)$

[5]

16. A person borrows £ 50 000 which is to be repaid over 20 years in 20 equal annual amounts. The lender charges an annual compound interest rate of 5% which remains constant throughout the period of the loan. The interest is added at the end of each year.

Each instalment is paid **after** the interest has been added to the account, with the first instalment paid on year after the loan was taken out.

Calculate the amount of each annual instalment.

[6]

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