GCE A LEVEL MARKING SCHEME

A LEVEL
MATHEMATICS – UNIT 3
1300U30 – 1

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About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

A2 Mathematics Unit 3: Pure Mathematics B
General instructions for marking GCE Mathematics

1. The mark scheme should be applied precisely and no departure made from it. Marks should be awarded directly as indicated and no further subdivision made.

2. Marking Abbreviations
   The following may be used in marking schemes or in the marking of scripts to indicate reasons for the marks awarded.

   cao = correct answer only
   MR = misread
   PA = premature approximation
   bod = benefit of doubt
   oe = or equivalent
   si = seen or implied
   ISW = ignore subsequent working

   F.T. = follow through ($\surd$ indicates correct working following an error and $\surd\!\!\!\times$ indicates a further error has been made)

   Anything given in brackets in the marking scheme is expected but, not required, to gain credit.

3. Premature Approximation
   A candidate who approximates prematurely and then proceeds correctly to a final answer loses 1 mark as directed by the Principal Examiner.

4. Misreads
   When the data of a question is misread in such a way as not to alter the aim or difficulty of a question, follow through the working and allot marks for the candidates' answers as on the scheme using the new data.
   This is only applicable if a wrong value, is used consistently throughout a solution; if the correct value appears anywhere, the solution is not classed as MR (but may, of course, still earn other marks).

5. Marking codes
   - ‘M' marks are awarded for any correct method applied to appropriate working, even though a numerical error may be involved. Once earned they cannot be lost.
   - ‘m’ marks are dependant method marks. They are only given if the relevant previous ‘M’ mark has been earned.
   - ‘A' marks are given for a numerically correct stage, for a correct result or for an answer lying within a specified range. They are only given if the relevant M/m mark has been earned either explicitly or by inference from the correct answer.
   - 'B' marks are independent of method and are usually awarded for an accurate result or statement.
   - ‘S’ marks are awarded for strategy
   - ‘E’ marks are awarded for explanation
   - ‘U’ marks are awarded for units
   - ‘P’ marks are awarded for plotting points
   - ‘C’ marks are awarded for drawing curves

Question			Marking Details	Marks Available
1	(a)	(i)	$\frac{dy}{dx} = 3(e^{-x} + x^2)^2 \cdot (-e^{-x} + 2x)$	2
		(ii)	$\frac{dy}{dx} = 2e^{2x} \sin 3x + e^{2x} \cdot 3\cos 3x$ $= e^{2x}(2\sin 3x + 3\cos 3x)$	3
		(iii)	$\frac{dy}{dx} = \frac{(-2\sin 2x)(x^2+1) - (\cos 2x)(2x)}{(x^2+1)^2}$	3
	(b)		$2 - 6xy^3 - 9x^2y^2\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{2 - 6xy^3}{-9x^2y^2}$	3

Question			Marking Details	Marks Available
2	(a)		Using $\cos 2x = 1 - 2\sin^2 x$: $2\sin^2 x - \sin x - 1 = 0$ $(2\sin x + 1)(\sin x - 1) = 0$ $\sin x = -\frac{1}{2} \quad \sin x = 1$ $x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$	6
	(b)		Using $\sec^2\theta = 1 + \tan^2\theta$: $2(1 + \tan^2\theta) - 5 = \tan\theta$ $2\tan^2\theta - \tan\theta - 3 = 0$ $(2\tan\theta - 3)(\tan\theta + 1) = 0$ $\tan\theta = \frac{3}{2} \quad \tan\theta = -1$ $\theta \approx 56.3^\circ, 135^\circ, 236.3^\circ, 315^\circ$	6

Question			Marking Details	Marks Available
3			$\cos\theta \approx 1 - \frac{\theta^2}{2}, \quad \sin 2\theta \approx 2\theta$ $0.2 - 1.6\theta - \frac{\theta^2}{2} \approx 0$ $\frac{1 - \frac{\theta^2}{2}}{1 + 2\theta} \approx 0.8$ $\theta^2 + 3.2\theta - 0.4 \approx 0$ Take the positive root: $\theta \approx 0.121$ (3 s.f.).	6

Question			Marking Details	Marks Available
4	(a)		$h = \frac{\pi/3-0}{4} = \pi/12$ Ordinates: $x = 0, \pi/12, \pi/6, \pi/4, \pi/3$ $y = \sec^2 x \Rightarrow y = [1, \approx 1.035, \approx 1.333, 2, 4]$ $\text{Area} \approx \frac{\pi}{12} \times \frac{1}{2}[1 + 4 + 2(1.035 + 1.333 + 2)]$ Answer: ≈ 1.799	4
	(b)		Using $\tan^2 x = \sec^2 x - 1$: $= \int (\sec^2 x + 1) dx$ $= 1.799 + \pi/3 = 2.846$	2

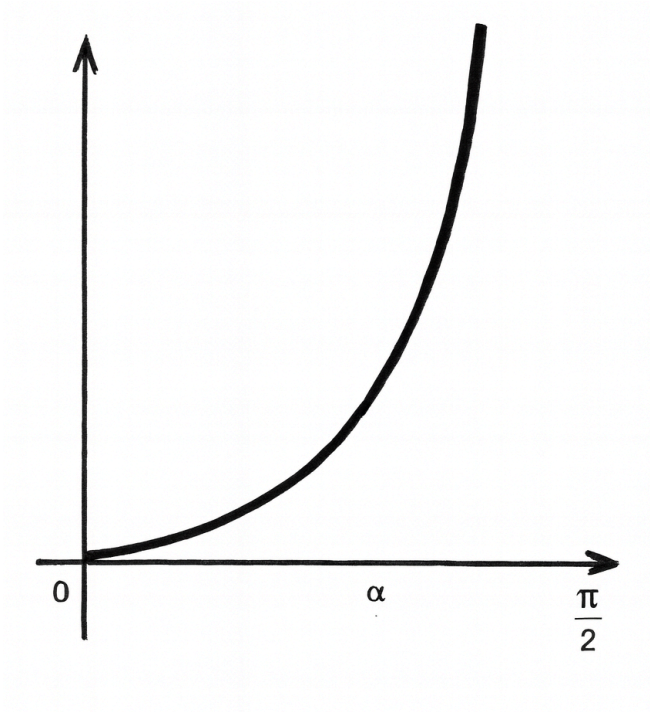
Question			Marking Details	Marks Available
5	(a)		$f'(x) = 3x^2 - 12x + 12$ $f'(x) = 0 \Rightarrow x = 2$ Point: $(2, 3)$ $f''(x) = 6x - 12 \;\; f''(2) = 0$ Test Nature f '' (1.9) = -0.6 f ' (2.1) = 0.6 There is a change of sign → Point of Infection	6
	(b)		Convex where $f''(x) > 0$: $x > 2$	2

Question			Marking Details	Marks Available
6	(a)		Iteration $x_{n+1} = \frac{1}{2}e^{x_n}$, $x_0 = 0.62$ $x_1 \approx 0.911$, $x_2 \approx 1.245$, $x_3 \approx 1.738$	2
	(b)	(i)	Newton-Raphson: $f(x) = e^x - 3x$, $f'(x) = e^x - 3$, $x_0 = 1.5$ $f(1.5) \approx 4.482 - 4.5 = -0.018$, $f'(1.5) \approx 4.482 - 3 = 1.482$ $x_1 = 1.5 - (-0.018)/1.482 \approx 1.512$	2
		(ii)	Check $f(1.5115)$ and $f(1.5125)$ show a sign change	3
		(iii)	Fails if $f'(x_0) = 0$. $f'(x) = e^x - 3$, so $f'(x) = 0$ when $x = \ln 3$ Derivative zero ⇒ method fails.	1

Question			Marking Details	Marks Available
7	(a)		Arithmetic series: $a = -3, l = 61, S_n = 957$ $S_n = \frac{n}{2}(a + l) \Rightarrow 957 = \frac{n}{2}(58) \Rightarrow n = 33$	2
	(b)		$l = a + (n - 1)d \Rightarrow 61 = -3 + 32d \Rightarrow d = 2$	2
	(c)		Middle term is 17th term: $T_{17} = a + 16d = -3 + 32 = 29$	1

Question			Marking Details	Marks Available
8			factor $(x-2)$ $= \frac{(x-2)(x-1)(x+3)(x+5)}{3(x+5)(x+3)(x-2)} = \frac{x-1}{3}$	5

Question			Marking Details	Marks Available
9			Assume $\exists$ integers x, y such that $3x + 15y = 2$. Left side divisible by 3: $3(x + 5y)$ Right side 2 not divisible by 3. Contradiction.	4

Question			Marking Details	Marks Available
10	(a)			1
	(b)		$\int_0^a \sec x dx > a$ because $\sec x > 1$ for $0 < x < a$, so area under $\sec x$ > area under $y = 1$	2

Question			Marking Details	Marks Available
11			Partial fractions: $\frac{5x+2}{(x+1)^2(2x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{2x-1}$ $A(x+1)(2x-1) = A(2x^2+x-1)$ $B(2x-1) = 2Bx - B$ $C(x+1)^2 = C(x^2+2x+1)$ Solving gives $A = -1, B = 1, C = 2$ $\int -\frac{1}{x+1}\,dx = -\ln(x+1)$ $\int \frac{1}{(x+1)^2}\,dx = -\frac{1}{x+1}$ $\int \frac{2}{2x-1}\,dx = \ln(2x-1)$ $F(2) - F(1) = \left(-\frac{1}{3}\right) - \left(-\ln 2 - \frac{1}{2}\right) = \ln 2 + \frac{1}{6}.$	10

Question			Marking Details	Marks Available
12			$= 1 + x + \frac{3}{4} \cdot 4x^2/2 + \left(-\frac{15}{8}\right)(-8x^3)/6$ $= 1 + x + \frac{3}{2}x^2 + \frac{15}{6}x^3 = 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3$ Valid for $\lvert -2x \rvert < 1 \Rightarrow \lvert x \rvert < \frac{1}{2}$	7

Question			Marking Details	Marks Available
13	(a)		$\frac{dx}{dt} = 6t^2, \frac{dy}{dt} = 6t$ $\frac{dy}{dx} = \frac{6t}{6t^2} = \frac{1}{t}$. At parameter p, gradient = $1/p$.	4
	(b)		At $t = 1$: point $(3, 3)$, gradient = 1 ⇒ normal gradient = -1. Equation: $y - 3 = -1(x - 3) \Rightarrow y = -x + 6$	4
	(c)		At $t = 0$: point $(1, 0)$, gradient undefined (vertical tangent). Equation: $x = 1$	3

Question			Marking Details	Marks Available
14	(a)		Integration by parts: $u = x^2, dv = \cos x dx$ $\int x^2 \cos x dx = x^2 \sin x - \int 2x \sin x dx$ $\int 2x \sin x dx = -2x \cos x + \int 2 \cos x dx = -2x \cos x + 2 \sin x$ So: $\int x^2 \cos x dx = x^2 \sin x + 2x \cos x - 2 \sin x$ Evaluate from 0 to $\pi/2$: $= \pi^2/4 - 2$	6
	(b)		$\int \frac{\ln x}{x(1+\ln x)^2} dx$, $u = 1 + \ln x$, $du = dx/x$ $\int \frac{u-1}{u^2} du = \int (1/u - 1/u^2) du = \ln \lvert u \rvert + 1/u + C$ $= \ln \lvert 1 + \ln x \rvert + \frac{1}{1 + \ln x} + C$	5

Question			Marking Details	Marks Available
15	(a)		As $x \to 3^+$, $f(x) \to +\infty$; as $x \to \infty$, $f(x) \to 0^+$. Range: $(0, \infty)$	2
	(b)		Let $y = \frac{9}{(x-3)^2} \Rightarrow (x-3)^2 = 9/y \Rightarrow x - 3 = 3/\sqrt{y}$ So $x = 3 + 3/\sqrt{y}$ $f^{-1}(x) = 3 + \frac{3}{\sqrt{x}}, x > 0$	5

Question			Marking Details	Marks Available
16			$50000 = P\left(\frac{1-(1.05)^{-20}}{0.05}\right)$ $P = \frac{50000 \times 0.05}{1-(1.05)^{-20}}$ $P \approx \frac{2500}{1-0.37689} \approx \frac{2500}{0.62311} \approx 4012.13$	6

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